PHY 835: Exercise 3
Released Feb 16, 2021;Target Due Date: March 2, 2021

1. MNIST – regression

The aim of this exercise is to get some experience with logistic regression and regularization. We shall use the implementations in sklearn for this exercise (Documentation for sklearn can be found online and it is useful to get familiar with the way this is written).

a) Using the SGCClassifier and the LogisticRegression, build classifiers for the MNIST dataset.
b) Find some handwritten digits which your classifiers are not capable of classifying correctly. How do they differ from correctly classified digits?
c) Scan over hyper-parameters (i.e. L1, L2 regularization, different optimizers) and compare the performance.
d) Compare your results to results you would have obtained by pure random guessing.

2. 1D Ising – Regression

In the lecture we have shown different performances of linear regression on predicting the energy in the 1D Ising model depending on our choice of regularization. The aim of this exercise is to analyze this performance a bit further.

a) Instead of the built-in scoring function, plot the error E_{out} for some choices of parameters.
b) How does the performance change when changing the number of samples?

3. Perceptron (XOR) and Shannon information content

a) Show that a single perceptron does not have the capability to realize the XOR function.
b) (Optional) How can this be avoided using multiple perceptrons?
c) For independent random variables ($P(X,Y) = P(X)P(Y)$) show that Shannon entropy is additive, i.e. $H(X, Y) = H(X) + H(Y)$.
d) (Optional) This is slightly off-topic but interesting nevertheless. Utilizing information content (e.g. by making it explicit on your decision tree), think about a solution for the following problem. You are given 12 balls, all equal in weight except for one that is either heavier or lighter. You are given a two-pan balance to use. In each use of the balance you may put any number of 12 balls on the left pan, and the same number on the right pan. Your task is to design a strategy to determine which is the odd ball and whether it is heavier or lighter than the others in as few uses of the balance as possible.